Course Code: 20HS0831 **R20**

SIDDHARTH INSTITUTE OF ENGINEERING & TECHNOLOGY:: PUTTUR   **(AUTONOMOUS)**   Siddharth Nagar, Narayanavanam Road – 517583   **QUESTION BANK (DESCRIPTIVE)**
Subject with Code: Differential Equation and Complex Analysis(20HS0831)   **Course & Branch:** B.tech - CE,EEE,ME,ECE & AGE
Regulation: R20   **Year & Sem:** I-B.Tech & II-Sem

UNIT –1
First and higher order Ordinary Differential Equations

1	a) Solve $(2x - y + 1)dx + (2y - x - 1)dy = 0$. b) Solve $(y^2 - 2xy)dx + (2xy - x^2)dy = 0$	[L3][CO1] [L6][CO1]	**[6M]** **[6M]**
2	a) Solve $\frac{dy}{dx} + \frac{ycosx+siny+y}{sinx+xcosy+x} = 0$ b) Solve $(x^2\text{-}ay)dx = (ax\text{-}y^2)dy$	[L3][CO1] [L3][CO1]	**[6M]** **[6M]**
3	a) Solve $x\frac{dy}{dx} + y = logx$. b)Solve $\frac{dy}{dx} + 2xy = e^{-x^2}$	[L6][CO1] [L3][CO1]	**[6M]** **[6M]**
4	a) Solve $(1 + y^2)dx = (tan^{-1}y - x)dy$ b)Solve $(x + 1)\frac{dy}{dx} - y = e^{3x}(x + 1)^2$	[L3][CO1] [L6][CO1]	**[6M]** **[6M]**
5	a) Solve $x\frac{dy}{dx} + y = x^3y^6$. b) Solve $\frac{dy}{dx} + y.tanx = y^2\, secx$	[L6][CO1] [L6][CO1]	**[6M]** **[6M]**
6	a) Solve $(D^2 + 5D + 6)y = e^x$ b)Solve $(D^2 - 4D + 3)y = 4e^{3x}$ given ; $y(0) = -1, y^1(0) = 3$	[L3][CO1] [L6][CO1]	**[6M]** **[6M]**
7	a) Solve $(D^2 - 3D + 2)y = cos3x$. b) Solve $(D^2 - 4D)y = e^x + sin3x.cos2x$	[L3][CO1] [L3][CO1]	**[6M]** **[6M]**
8	a) Solve $(D^2 + 4D + 4)y = 4cosx + 3sinx$ **b)** Solve $(D^2 + 1)y = sinx.sin2x$	[L6][CO1] [L3][CO1]	**[6M]** **[6M]**
9	a) Solve $(D^2 + 4)y = e^x + sin2x + cos2x$. **b)**Solve $(D^2 + D + 1)y = x^3$	[L6][CO1] [L6][CO1]	**[6M]** **[6M]**
10	a) Solve $(D^2 - 3D + 2)y = xe^{3x} + sin2x$ b) Solve $(D^2 + 4D + 3)y = e^{-x}sinx + x$.	[L3][CO1] [L6][CO1]	**[6M]** **[6M]**

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UNIT –2
Equations reducible to Linear Differential Equations

1	a) Solve $(D^2+a^2)y=\tan ax$ by the method of variation of parameters. b)Solve $(D^2-2D)y=e^x\sin x$ by the method of variation of parameters.	[L3][CO2] [L6][CO2]	**[6M]** **[6M]**
2	a) Solve $(D^2+4)y=Sec2x$ by the method of variation of parameters. b) Solve $(D^2+1)y=Co\sec x$ by the method of variation of parameters.	[L3][CO2] [L6][CO2]	**[6M]** **[6M]**
3	a)Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$. b) Solve $x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^4$	[L3][CO2] [L3][CO2]	**[6M]** **[6M]**
4	a)Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\,logx}{x^2}$ b) Solve $(x^2D^2-4xD+6)y=x^2$	[L6][CO2] [L3][CO2]	**[6M]** **[6M]**
5	Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=sin2[\log(1+x)]$	[L6][CO2]	**[12M]**
6	Solve $(1+x)^2\frac{d^2y}{dx^2}-3(1+x)\frac{dy}{dx}+4y=x^2+x+1$	[L3][CO2]	**[12M]**
7	a)Solve $\frac{dx}{dt}=3x+2y\ \ ;\frac{dy}{dt}+5x+3y=0.$ b) Solve $\frac{dy}{dx}+y=z+e^x\ \ ;\frac{dz}{dx}+z=y+e^x.$	[L3[CO2] [L6][CO2]	**[6M]** **[6M]**
8	Solve $\frac{dx}{dt}+2x+y=0\ \ ;\frac{dy}{dt}+x+2y=0; given\ x=1\ and\ y=0\ when\ t=0$	[L6][CO2]	**[12M]**
9	An uncharged condenser of capacity is charged applying an e.m.f $E\ sin\frac{t}{\sqrt{LC}}$ through leads of self-inductance L and negligible resistance. Prove that at time ‘t’ the charge on one of the plates is $\frac{EC}{2}\left[sin\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}cos\frac{t}{\sqrt{LC}}\right]$.	[L5][CO2]	**[12M]**
10	Find the current ‘i’ in the LCR circuit assuming zero initial current and charge q. If R=80 ohms, L=20 henrys, C=0.01 farads and E=100 V.	[L1][CO2]	**[12M]**

UNIT-3
Partial Differential Equations

1	a)Form the partial differential equation by eliminating the constants from $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$. b)Form the partial differential equation by eliminating the constants from $(x-a)^2 + (y-b)^2 = z^2 cot^2\alpha$ where 'α' is a parameter	[L2][CO3] [L2][CO3]	**[6M]** **[6M]**
2	a)Form the partial differential equation by eliminating the constants from $z = a.\log\left[\frac{b(y-1)}{(1-x)}\right]$. b)Form the partial differential equation by eliminating the constants from $\log(az-1) = x + ay + b$.	[L2][CO3] [L2][CO3]	**[6M]** **[6M]**
3	a) Form the partial differential equation by eliminating the arbitrary functions from $z = f(x^2 - y^2)$. b) Form the partial differential equation by eliminating the arbitrary functions from $\mathbf{z = f(x) + e^y . g(x)}$	[L2][CO3] [L2][CO3]	**[6M]** **[6M]**
4	a) Form the partial differential equation by eliminating the arbitrary function from $xyz = f(x^2 + y^2 + z^2)$ b) Form the partial differential equation by eliminating the arbitrary function from $z = xy + f(x^2 + y^2)$	[L2][CO3] [L2][CO3]	**[6M]** **[6M]**
5	a) Form the P.D.E by eliminating the arbitrary function from $\emptyset\left(\frac{y}{x}, x^2 + y^2 + z^2\right) = 0$.. b) Form the P.D.E by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.	[L2][CO3] [L2][CO3]	**[6M]** **[6M]**
6	a) Solve by the method of separation of variables $u_x = 2u_y + u, where\ u(x,0) = 6e^{-3x}$? a)Solve by the method of separation of variables $4u_x + u_y = 3u, given\ \ u(0,y) = e^{-5y}$	[L3][CO3] [L6][CO3]	**[6M]** **[6M]**
7	a) Solve by the method of separation of variables $3u_x + 2u_y = 0, where\ u(x,0) = 4e^{-x}$ b) Solve by the method of separation of variables $u_x - 4u_y = 0, where\ u(0,y) = 8e^{-3y}$	[L3][CO3] [L6][CO3]	**[6M]** **[6M]**
8	A tightly stretched string with fixed end points $x = 0\ and\ x = l$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each of its points a velocity $kx(l-x)$ find the displacement of the string at any distance from one end at any time t.	[L1][CO3]	**[12M]**
9	Find the temperature $u(x,t)$ in a bar OA of length l which is perfectly insulated laterally and whose ends O and A are kept at0°C ,given that the initial temperature at any point P of the rod (where OP=x) is given as $u(x,o) = f(x), (0 \le x \le l)$.	[L1][CO3]	**[12M]**
10	Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \mathbf{0\ with\ u(0,y) = 0 = u(x,0), u(l,y) = 0\ and}$ $\mathbf{u(x,a) = sin(\frac{n\pi x}{l})}$	[L3][CO3]	**[12M]**

UNIT-4
COMPLEX VARIABLE- DIFFERENTATION

1	Show that $u(x,y)=e^{2x}\left(x\cos 2y - y\sin 2y\right)$ is harmonic and find its harmonic conjugate.	[L1,L2][CO4]	**[12M]**
2	a) Show that $u = \frac{1}{2}log(x^2 + y^2)$ is harmonic. b) If $W = f(z)$is analytic function then prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left\|\text{Re}\,al f(z)\right\|^2 = 2\left\|f^1(z)\right\|^2$	[L2][CO4] [L5][CO4]	**[6M]** **[6M]**
3	a) Find 'a' and 'b' if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic.Hence find $f(z)$ $interms\ of\ z$ b) Find the analytic function whose imaginary part is $e^x\left(x\sin y + y\cos y\right)$.	[L1][CO4] [L1][CO4]	**[6M]** **[6M]**
4	a) Determine p such that the function $f(z)=\frac{1}{2}\log\left(x^2+y^2\right)+i\tan^{-1}\left(\frac{px}{y}\right)$ is analytic b) Find all the values of k, such that$f(z) = e^x\,(\cos ky + i\sin ky)$ is analytic.	[L5][CO4] [L1][CO4]	**[6M]** **[6M]**
5	a) If $f(z)=u+iv$ is an analytic function of z and if $u+v=e^x\left(\cos y - \sin y\right)$, Find $f(z)$in terms of z. b) Find an analytic function whose real part is $e^{-x}\left(x\sin y - y\cos y\right)$.	[L1][CO4] [L1][CO4]	**[6M]** **[6M]**
6	a) Show that f(z) = $z + 2\bar{z}$ is not analytic anywhere in the complex plane. b) Show that $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=4\frac{\partial^2}{\partial z\partial\bar{z}}$.	[L2][CO4] [L2][CO4]	**[6M]** **[6M]**
7	a) Find the bilinear transformation which maps the points (∞, ,0) into the points$(0,i,\infty)$. b)Find the image of the triangular region with vertices at $(0,0)(1,0)(0,1)$ under the transformation $w = (1 - i)z + 3$.	[L1][CO4] [L1][CO4]	**[6M]** **[6M]**
8	a) Find the image of the infinite strip $0 < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$ b)Show that the function $w = \frac{4}{z}$ transforms the straight line $x = c$ in the z-plane into a circle in the $w - plane$.	[L1][CO4] [L2][CO4]	**[6M]** **[6M]**
9	a) Find the bilinear transformation which maps the points (∞, i, 0) into the points (−1, −i, 1) in w-plane. b)Find the bilinear transformation that maps the points(1, i, −1) into the points (2, i, −2) in w-plane.	[L1][CO4] [L1][CO4]	**[6M]** **[6M]**
10	a) Find the image of infinite strip bounded by $x=0\ \&\ x=\frac{\pi}{4}$ under the transformation $w = \cos z$. b) Prove that the transformation $w = \sin z$maps the families of lines $x = y = constant$ into two families of confocal central conics.	[L1][CO4] [L5][CO4]	**[6M]** **[6M]**

UNIT-5
COMPLEX VARIABLE- INTEGRATION

1	Show that $\int_c (z+1)dz = 0$ where C is the boundary of the square whose vertices at the points $z = 0, z = 1, z = 1+i, z = i$	[L2][CO5]	**[12M]**
2	a) Evaluate the line integral $\int_c (y - x - 3x^2 i)dz$ where c consists of the line segments from $z=0$ to $z=i$ and the other from $z=i$ to $z=i+1$. b) Evaluate $\int_0^{1+3i} (x^2 - iy)\,dz$ along the path $y = x$.	[L5][CO5] [L5][CO5]	**[6M]** **[6M]**
3	Verify Cauchy's theorem for the function $f(z) = 3z^2 + iz - 4$ if c is the square with vertices at $1 \pm i$ and $-1 \pm i$	[L6][CO5]	**[12M]**
4	a) Evaluate using Cauchy's integral formula $\int_c \frac{\sin^6 z}{\left(z - \frac{\pi}{2}\right)^3} dz$ around the circle $c : \lvert z \rvert = 1$. b) Evaluate $\int_c \frac{\log z \, dz}{(z-1)^3}$ where $c : \lvert z - 1 \rvert = \frac{1}{2}$ using Cauchy's integral formula.	[L5][CO5] [L5][CO5]	**[6M]** **[6M]**
5	a) Expand $f(z) = \sin z$ in Taylor's series about $z = \frac{\pi}{4}$. b) Expand $f(z) = \log z$ in Taylor's series about $z = 1$.	[L2][CO5] [L2][CO5]	**[6M]** **[6M]**
6	a) Find the Laurent's series expansion of the function $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < \lvert z + 2 \rvert < 5$. b) Find the Laurent's series of the function $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$.	[L1][CO5] [L1][CO5]	**[6M]** **[6M]**
7	a) Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$ and the residues at each pole. b) Find the residue of the function $f(z) = \frac{1}{(z^2+4)^2}$ where c is $\lvert z - i \rvert = 2$	[L5][CO5] [L1][CO5]	**[6M]** **[6M]**
8	a) Evaluate $\int_c \frac{dz}{z^3 (z+4)}$ where c is $\lvert z \rvert = 2$. b) Determine the poles and residues of $tanhz$.	[L5][CO5] [L5][CO5]	**[6M]** **[6M]**
9	Evaluate $\int_0^{2\pi} \frac{1}{a + b\cos\theta} d\theta = \frac{\pi}{\sqrt{a^2 - b^2}}, a > b > 0$	[L5][CO5]	**[12M]**
10	Show that $\int_0^{2\pi} \frac{d\theta}{1 + a^2 - 2a\cos\theta} = \frac{2\pi}{1 - a^2}$, $0 < a < 1$	[L2][CO5]	**[12M]**